MATHEMATICS

Chapter 6: LINEAR INEQUALITIES

LINEAR INEQUALITIES

Some Important Results

1. If ‘k’ is a positive real number, then $|x| < k \Leftrightarrow -k < x < k$, $x \in (-k, k)$.

2. If ‘k’ is a positive real number, then $|x| \le k \Leftrightarrow -k < x < k$, $x \in (-k, k)$.

3. If ‘k’ is a positive real number, then $|x| > k \Leftrightarrow x < -k, x > k$.

4. If ‘k’ is a positive real number, then $|x| \ge k \Leftrightarrow x \le -k, x \ge k$.

5. If ‘k’ is a positive real number and ‘y’ is a fixed number, then
   $|x - y| < k \Leftrightarrow y - k < x < y + k$, $x \in (y - k, y + k)$.

6. If ‘k’ is a positive real number and ‘y’ is a fixed number, then
   $|x - y| \le k \Leftrightarrow y - k \le x \le y + k$, $x \in [y - k, y + k]$.

7. If ‘k’ is a positive real number and ‘y’ is a fixed number, then
   $|x - y| > k \Leftrightarrow x < y - k$ or $x > y + k$

8. If ‘k’ be a positive real number and ‘y’ is a fixed number, then
   $|x - y| \ge k \Leftrightarrow x \le y - k$ or $x \ge y + k$

9. If a, b and c are positive real numbers, then $a < |x| < b \Leftrightarrow x \in (-b, a) \cup (a, b)$

10. If a, b and c are positive real numbers, then $a \le |x| \le b \Leftrightarrow x \in [-b, a] \cup (a, b)$

11. If a, b and c are positive real numbers, then $a \le |x - c| \le b \Leftrightarrow x \in [-b + c, -a + c] \cup [a + c, b + c]$

12. If a, b and c are positive real numbers, then $a < |x - c| < b \Leftrightarrow x \in (-b + c, -a + c) \cup (a + c, b + c)$

Definitions

1. Two real numbers or two algebraic expressions related by the symbol ‘<’, ‘>’, ‘≤’ or ‘≥’ form aninequality.
2. Inequalities containing ‘<’, or ‘>’ are called strict inequalities.
3. Inequalities containing ‘≤’ or ‘≥’ are called slack inequalities.
4. An inequality containing any two of ‘<’, ‘>’, ‘≤’ or ‘≥’ is called double inequality.
5. Solution an inequality in one variable is the value of the variable which makes it a true statement.
6. Solving an inequality is the process of getting all possible solutions of an inequality.

7. Solution set is the set of all possible solutions of an inequality is known as its solution set.
8. A linear expression in one variable involving the inequality symbol is linear inequality in one variable.

   General forms

   $ax + b < 0$

   $ax + b > 0$

   $ax + b \leq 0$

   $ax + b \geq 0$

9. A linear inequality involving two variables is known as a linear inequality in two variables. Generalforms

   $ax + by < c$

   $ax + by > c$

   $ax + by \leq c$

   $ax + by \geq c$

10. The following is the example of quadratic inequalities known as quadratic inequalities

    $ax^2 + bx + c \leq 0$

    $ax^2 + bx + c \geq 0$

11. The inequalities of the form '<' or '>' are known as strict inequalities, whereas if they are of the form ≤ or ≥ then they are called slack inequalities.
12. The region containing all the solutions of an inequality is called the solution region.
13. The solution region of the system of inequalities is the region which satisfies all the given inequalitiesin the system simultaneously.
14. Quadratic inequality is the quadratic polynomial with an inequality sign. Generic quadratic inequality isof the form $ax^2 + bx + c > 0$.

Concepts

1. If two real numbers are related by the symbols '<', '>', '≤' or '≥', then the inequality is a numericalinequality and in case of algebraic expressions it is literal inequality.

   $2<3$ is numerical inequality.

   $5x + 2 \leq 7$ is literal inequality.

2. **Rules for simplifying the inequalities**

Rule 1: Equal numbers may be added to (or subtracted from) both sides of an equation.

If $a < b$, then $a + c < b + c$.

Rule 2: Both sides of an equation may be multiplied (or divided) by the same non-zero number.
If $a < b$, then $ac < bc$

Rule 3: Sign of inequality is reversed in case of multiplication (or division) by a negative number.
If $a < b$, then $ak > bk$, where k is a negative number.

Rule 4: Sign of inequality is reversed in case of taking the reciprocals.

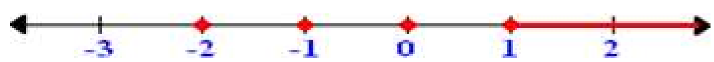
3. A linear inequality can be solved by the following steps:

1. Obtain the linear inequality.
2. Group all variable terms on one side of the inequality and transpose the constant termon the other side of the inequality.
3. Simplify both the sides of the inequality to the standard form
   $ax < b$, or $ax \leq b$, or $ax > b$, or $ax \geq b$
4. Solve the inequality by dividing both the sides of the inequality by the coefficient of the variable.
5. Write and depict the solution set in the form of number line.

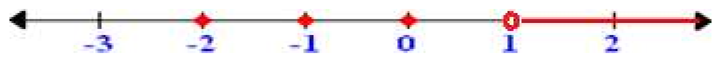
4. **Some examples of graphs of linear inequality:** A linear inequality in one variable can be represented graphically as follows:

Representation of $x \leq 1$

Representation of $x \geq 1$



Representation of x > 1



5. Steps to solve inequalities of the type, $\frac{ax+b}{cx+d} > k, \frac{ax+b}{cx+d} \geq k, \frac{ax+b}{cx+d} < k$ and $\frac{ax+b}{cx+d} \leq k$

   Step 1: Transpose all the variable terms on LHS.

   Step 2: Transpose all the variable terms on LHS.

   Step 3: Simplify the terms in LHS.

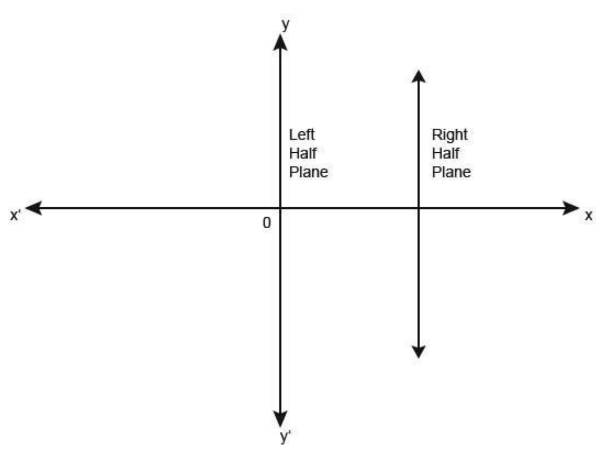
   Step 4: Make the coefficient of x positive in the numerator and denominator.

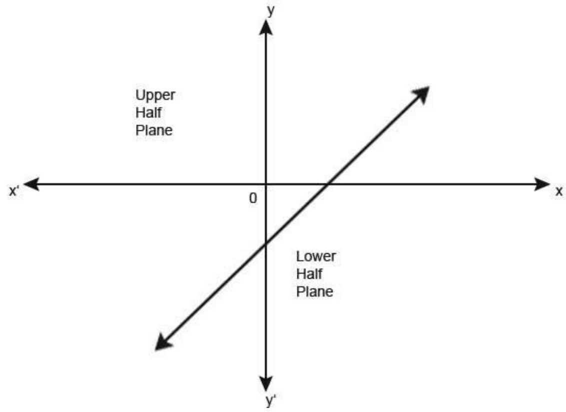
   Step 5: Equate both the numerator and denominator to zero and find the values of x and critical points.

   Step 6: Plot these critical points on the number line, which will divide the real line into three regions.
   Step 7: Mark the sign of the functions over the respective intervals.

   Step 8: Write solution regions in the form of intervals to get the required solution sets of the giveninequality.

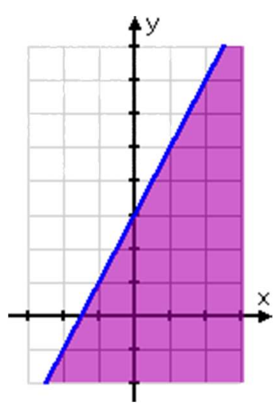
6. A linear inequality in two variables represents a half plane geometrically. Types of half planes.





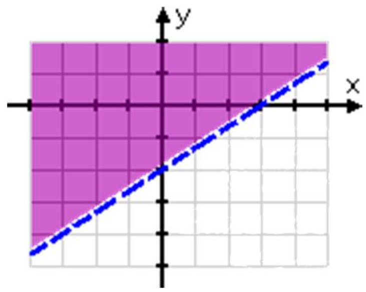
7. In order to identify the half plane represented by an inequality, take any point (a, b) (not on line) and check whether it satisfies the inequality or not. If it satisfies the inequality, then the inequality represents the half plane and subsequently shades the region which contains the point; otherwise, the inequality represents the half plane which does not contain the point within it. For convenience, the point (0, 0) is preferred.

8. If an inequality is of the type ax + b ≥ c or ax + b ≤ c, i.e slack inequality, then the points on the line ax+ b = c are also included in the solution region.



Solution of slack inequality

9. If an inequality is of the form ax + by > c or ax + by < c, then the points on the line ax + by = c are notto be included in the solution region.



Solution of strict inequality

10. To represent $x < a$ (or $x > a$) on a number line, put a circle on the number a and dark line to the left (orright) of the number a.

11. To represent $x \leq a$ (or $x \geq a$) on a number line, put a dark circle on the number a and dark line to theleft (or right) of the number x.

12. Steps to represent the linear inequality in two variables graphically

    Step 1: Rewrite the inequality as linear equation, i.e. $ax + by = c$.

    Step 2: Put $x = 0$ to get y-intercept of the line, i.e. $(0, c/b)$.

    Step 3: Put $y = 0$ to get x-intercept of the line, i.e. $(c/a, 0)$.

    Step 4: Join the two points, each on X-axis and Y-axis to get the graphical representation of theline.

    Step5: Choose a point (x_1, y_1) in one of the planes, i.e. either to the left or right or upper or lower half of the line, but not on the line.

    Step 6: If (x_1, y_1) satisfies the given inequality, then the required region is that particular half plane in which (x_1, y_1) lie.

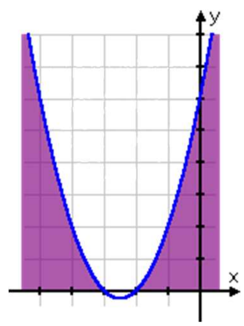
    On the other hand, if (x_1, y_1) does not satisfy the given inequality, then the required solution region is the half plane which does not contain (x_1, y_1).

13. Linear inequalities represent regions; regions common to the given inequalities will be the solutionregion.
    Similar to linear equations, there can be cases of overlapping of regions or no common regions for thegiven inequalities.

14. To solve a system of inequalities graphically
    - Change the sign of equality to inequality and draw the graph of each line.
    - Shade the region for each inequality.
    - Common region to all the inequalities is the solution.

15. A linear inequality divides the plane into two half planes, while a quadratic inequality is represented bya parabola which divides the plane into different regions.

    Region represented by the inequality $x^2 + 5x + 6 \geq 0$



16. Interval Notations

Open Interval: The interval which contains all the elements between a and b excluding a and b**.** In set notations:

$(a, b) = \{x : a < x < b\}$

Closed interval: The interval which contains all the elements between a and b and also the end points aand b is called the **closed interval.**

$[a, b] = \{x : a \leq x \leq b\}$

Semi open intervals:

$(a, b) = \{x : a \leq x < b\}$ includes all the elements from a to b including a and excluding b.

$[a, b] = \{x : a < x \leq b\}$ includes all the elements from a to b excluding a and including b.

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 6

Linear Inequalities

Notations

- The notation a<b means that 'a is less than b'.
- The notation a>b means that 'a is greater than b'.
- The notation a≥b means that 'a is greater than or equal to b'.
- The notation a≤b means that 'a is less than or equal to b'.
- The notation a≠b means that 'a is not equal to b'.

Solution of Linear Inequality in One Variable

Solution and Solution set

Solution: Values of x , which make inequality a true statement.

e.g.: '3' is the solution for x<7.

Solution Set:

The set of values of x is called its solution set.

e.g.: {1,2,3,4,5,6} is solution set for x<7 , where x is a natural number.

Types of Inequalities

Numerical Inequalities : Relation between numbers e.g.: 3<5, 4>7

Literal or Variable Ineqalities : Relation between two variables or variable and numbers. e.g.:y<8

Double Inequalities: Relationship from two sides e.g.:2<x<5

Strict Inequalities: An inequality that uses the symbol <or> e.g.: x<5, 3<5

Slack Inequality: An inequality that uses the symbols ≤ or ≥. e.g.: y≤5

Linear Inequalities in One Variable: An inequality which involves a linear function in one variable e.g.: x<5

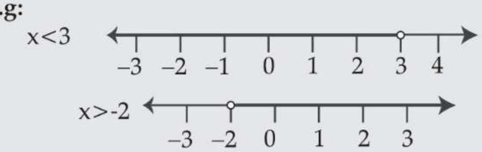
Linear Inequalities in two variables: An inequality which involves a linear function in two variables, e.g.: 3x+2y<5

Quadratic Inequalities: An inequality which involves a quadratic function, eg.: $x^2+2x \le 5$.

Representation

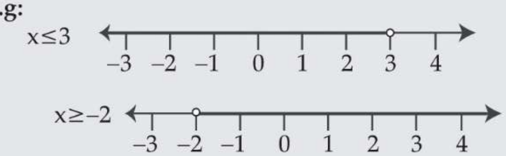
To represent x<a (x>a) on a number line, put a circle on the number 'a' and dark line to the left (or right) of the number 'a'.

e.g:



To represent x≤a (or x≥a) on a number line, put a dark circle on the number 'a' and dark the line to the left (or right) of the number 'a'.

e.g:



Conditions

- An inequality is an relation that holds between two values, when they are different eg.: x<5, here, there is a relation between x & 5.
- Two real numbers or algebraic expressions related by symbol '<', '>', '≤' or '≥' form inequality. eg,: y≤4, 2x+3y≥5, 3>x.

Rules of Inequality

- Equal numbers may be added to (or subtracted from) both the sides of an inequality without affecting the sign of inequality:
  e.g: x<7 is same as x+2 < 7+2
- Both sides of an inequality can be multiplied (or divided) by the same positive number without affecting the sign of inequality.
  e.g: x+y< 7 is same as (x+y) × 3<7 × 3
- When both sides are multiplied or divided by a – ve number, then sign of inequality is reversed.
  e.g: x+y< 7 is same as (x+7) × (–3) > 7 × (–3)

Graphical Solution of Linear Inequalities in Two Variables

- A line divides the cartesian plane into two equal parts. A point in cartesian plane will either lie on a line or will lie in either of half planes. The region containing all the solutions of inequality is called the solution region.
- In order to identify the half plane reprsented by an inequality, it is just sufficient to take any point (a, b) (not on line) and check whether it satisfies the inequality or not. If it satisifes, then inequality represents the half plane and shade the region, which contain the point otherwise the inequality repersents the half plane which does not contain the point within it.

Important Questions

Multiple Choice questions-

Question 1. If $-2 < 2x - 1 < 2$ then the value of x lies in the interval

(a) (1/2, 3/2)

(b) (-1/2, 3/2)

(c) (3/2, 1/2)

(d) (3/2, -1/2)

Question 2. If $x^2 < -4$ then the value of x is

(a) (-2, 2)

(b) (2, ∞)

(c) (-2, ∞)

(d) No solution

Question 3. If $|x| < -5$ then the value of x lies in the interval

(a) (-∞, -5)

(b) (∞, 5)

(c) (-5, ∞)

(d) No Solution

Question 4. The graph of the inequations $x \leq 0$, $y \leq 0$, and $2x + y + 6 \geq 0$ is

(a) exterior of a triangle

(b) a triangular region in the 3rd quadrant

(c) in the 1st quadrant

(d) none of these

Question 5. The graph of the inequalities $x \geq 0$, $y \geq 0$, $2x + y + 6 \leq 0$ is

(a) a square

(b) a triangle

(c) { }

(d) none of these

Question 6. Solve: $2x + 1 > 3$

(a) [-1, ∞]

(b) (1, ∞)

(c) (∞, ∞)

(d) (∞, 1)

Question 7. The solution of the inequality 3(x – 2)/5 ≥ 5(2 – x)/3 is

(a) x ∈ (2, ∞)

(b) x ∈ [-2, ∞)

(c) x ∈ [∞, 2)

(d) x ∈ [2, ∞)

Question 8. Solve: 1 ≤ |x – 1| ≤ 3

(a) [-2, 0]

(b) [2, 4]

(c) [-2, 0] ∪ [2, 4]

(d) None of these

Question 9. Solve: -1/(|x| – 2) ≥ 1 where x ∈ R, x ≠ ±2

(a) (-2, -1)

(b) (-2, 2)

(c) (-2, -1) ∪ (1, 2)

(d) None of these

Question 10. If $x^2 < 4$ then the value of x is

(a) (0, 2)

(b) (-2, 2)

(c) (-2, 0)

(d) None of these

Very Short:

1. Solve $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$

2. Solve 3x + 8 > 2 when x is a real no.

3. Solve the inequality $\frac{x}{4} < \frac{(5x-2)}{3}\frac{(7x-3)}{5}$

4. **If** $4x > -16$ **then** $x \square -4.$

5. Solve the inequality $\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x - 6)$

6. Solution set of the in inequations 2x – 1 ≤ 3 and 3x + 1 ≥ -5 is.
7. Solve. 7x + 3 < 5x + 9. Show the graph of the solution on number line.
8. Solve the inequality. $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{2-x}{5}$
9. Solve 5x – 3 $\leq 3x + 1$ when x is an integer.
10. Solve 30x < 200 when x is a natural no.

Short Questions:

1. Solve 3x – 6 ≥ 0 graphically
2. Ravi obtained 70 and 75 mark in first unit test. Find the minimum marks he should get in the third test to have an average of at least 60 marks.
3. Ravi obtained 70 and 75 mark in first unit test. Find the minimum marks he should get in the third test to have an average of at least 60 marks.
4. A company manufactures cassettes and its cost equation for a week is C=300+1.5and its revenue equation is R = 2 x, where x is the no. of cassettes sold in a week. How many cassettes must be sold by the company to get some profit?
5. The longest side of a Δ is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the Δ is at least 61 cm find the minimum length of the shortest side.

Long Questions:

1. IQ of a person is given by the formula $IQ = \frac{MA}{CA} \times 100$

   Where MA is mental age and CA is chronological age. If 80 ≤ IQ ≤ 140 for a group of 12yr old children, fond the range of their mental age.
2. Solve graphically 4x + 3y ≤ 60 $y \geq 2x$ $x \geq 3$ $x, y \geq 0$
3. A manufacturer has 600 liter of a 12% sol. Of acid. How many liters of a 30% acid sol. Must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%.
4. Solve graphically $x - 2y \leq 3$ $3x + 4y \geq 12$ $x \geq 0$ $y \geq 1$
5. A sol. Of 8% boric acid is to be diluted by adding a 2% boric acid sol. to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 liters of the 8% sol. how many liter of the 2% sol. will have to be added.

Answer Key:

MCQ:

1. (b) (-1/2, 3/2)
2. (d) No solution
3. (d) No Solution
4. (b) a triangular region in the 3rd quadrant
5. (c) { }
6. (b) (1, ∞)
7. (d) x ∈ [2, ∞)
8. (c) [-2, 0] ∪ [2, 4]
9. (c) (-2, -1) ∪ (1, 2)
10. (b) (-2, 2)

Very Short Answer:

1.

$$\frac{3x-4}{2} \geq \frac{x+1}{4} - \frac{1}{1}$$

$$\frac{3x-4}{2} \geq \frac{x+1-4}{4}$$

$$\frac{3x-4}{2} \geq \frac{x-3}{4}$$

$$2(3x-4) \geq (x-3)$$

$$6x-8 \geq x-3$$

$$x \geq 1$$

2.

$$3x+8 > 2$$

$$3x > 2-8$$

$$3x > -6$$

$$x > -2$$

$$(-2, \infty)$$

3.

$$\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$$

$\frac{x}{4} < \frac{5(5x-2)-3(7x-3)}{15}$

$\frac{x}{4} < \frac{4x-1}{15}$

$15x < 16x - 4$

$-x < -4$

$x > 4$

$(4, \infty)$

4. x > -4.

5.

$\frac{1}{2}\left(\frac{3x}{5}+4\right) \geq \frac{1}{3}(x-6)$

$\frac{3x}{10}+2 \geq \frac{x}{3}-2$

$\frac{3x}{10}-\frac{x}{3} \geq -4$

$\frac{9x-10x}{30} \geq -4$

$\frac{-x}{30} \geq -4$

$-x \geq -120$

$x \leq 120$

$(-\infty, 120]$

6.

$2x-1 \leq 3\,,\ \ 3x+1 \geq -5$

$\Rightarrow 2x \leq 4,\ \ 3x \geq -6$

$\Rightarrow x \leq 2,\ \ x \geq -2$

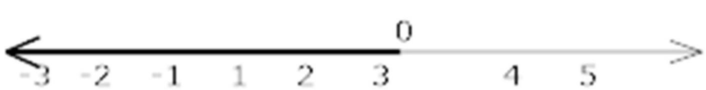
$\Rightarrow -2 \leq x \leq 2$

7.

$7x+3 < 5x+9$

$2x < 6$

$x < 3$



8.

$$\frac{2x-1}{3} \geq \frac{5(3x-2)-4(2-x)}{20}$$

$$20(2x-1) \geq 3(19x-18)$$

$$40x-20 \geq 57x-54$$

$$-17x \geq -34$$

$$x \leq 2$$

$$(-\infty, 2]$$

9.

$$5x-3 \leq 3x+1$$

$$5x-3x \leq 4$$

$$2x \leq 4$$

$$x \leq 2$$

$$\{\ldots, -3, -2, -1, 0, 1, 2\}$$

10.

$$30x < 200$$

$$x < \frac{200}{30}$$

$$x < \frac{20}{3}$$

Solution set of the inequality $\{1, 2, 3, 4, 5, 6\}$

Short Answer:

1.

$$3x-6 \geq 0 \ldots\ldots(i)$$

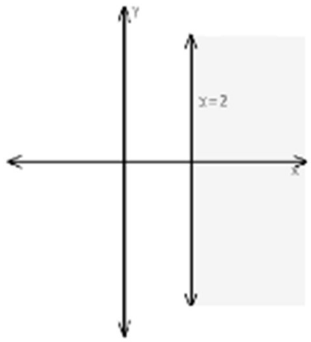
$$3x-6=0$$

$$x=2$$

Put (0,0) in eq. (i)

$$0-6 \geq 0$$

$0 > 6$ false.



2.

Let Ravi secure x marks in third test

ATQ $\frac{70+75+x}{3} \geq 60$

$x \geq 135$

3.

Let x and $x+2$ be consecutive odd natural no.

ATQ $x > 10.......(i)$

$(x)+(x+2) < 40$

$x < 19......(ii)$

From (i) and (ii)

$(11,13)$ $(13,15)$, $(15,17)$ $(17,19)$

4.

Profit = revenue – cost

$R > C$ [for to get some profit]

$2x > 300 + 1.5x$

$\frac{1}{2}x > 300$

$x > 600$

5.

Let shortest side be x cm then the longest side is 3x cm and the third side (3x -2) cm.

ATQ (x) + (3x) + (3x - 2) ≥ 61

$x \geq 9$

Length of shortest side is 9 cm.

Long Answer:

1.

$80 \le IQ \le 140$ (Given)

$80 \le \frac{MA}{CA} \times 100 \le 140$

$80 \le \frac{MA}{12} \times 100 \le 140$

$80 \times \frac{12}{100} \le MA \times \frac{100}{12} \times \frac{12}{100} \le 140 \times \frac{12}{100}$

$\frac{96}{10} \le MA \le \frac{168}{10}$

$9.6 \le MA \le 16.8$

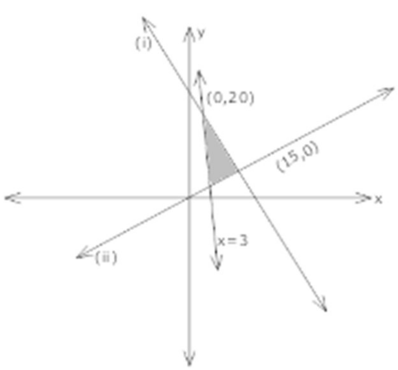
2. 4x + 3y = 60

x	**0**	**15**
y	20	0

$y = 2x$

x	**0**	**20**
y	0	40

$x = 3$



3.

Let x litres of 30% acid sol. Is required to be added.

$30\%x+12\%$ of $600>15\%$ of $(x+600)$ and

$30\%x+12\%$ of $600<18\%$ of $(x+600)$

$$\frac{30x}{100}+\frac{12}{100}(600)>\frac{15}{100}(x+600)$$

$$\frac{30x}{100}+\frac{12}{100}(600)<\frac{18}{100}(x+600)$$

$x>120$ and $x<300$

i.e. $120<x<300$.

4.

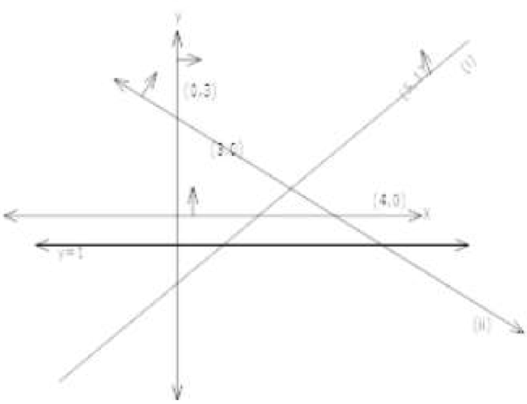
$x-2y=3$

x	**3**	**5**
y	0	1

$3x+4y=12$

x	**0**	**4**
y	3	0

$y=1$



5.

Let x be added

ATQ 2% of $x+8\%$ of $640 > 4\%$ of $(640+x)$

$$\frac{2x}{100}+\frac{8\times640}{100}>\frac{4}{100}(640+x)$$

$x<1280$.......(i)

And 12% of $x+8\%$ of $640<6\%$ of $(640+x)$

$$\frac{2x}{100}+\frac{8\times640}{100}<\frac{6}{100}(640+x)$$

$x>320$.......(ii)

From (i) and (ii)

$320<x<1280$